ERRATA
MATHEMATICS FOR THE INTERNATIONAL STUDENT
MATHEMATICS HL (Core) (3rd edition)

Third edition - 2018 fourth reprint

The following erratum was made on 21/Feb/2020

page 955  ANSWERS EXERCISE 27A 79, should read:

79  \( x = -\frac{11n}{12'}, -\frac{7n}{12'}, \frac{5n}{12'} \)
ERRATA
MATHEMATICS FOR THE INTERNATIONAL STUDENT
MATHEMATICS HL (Core) (3rd edition)

Third edition - 2017 third reprint

The following erratum was made on 25/Jul/2017

page 623  REVIEW SET 20A 10 b, should read:
10  b Show that if $A\overline{PM}=B\overline{PM}$, then the length of cable is given by
$L(\theta) = 2 \csc \theta + 3 - \cot \theta$ km.

The following erratum was made on 30/Jan/2017

page 863  EXERCISE 27B 98, should read:
98 Suppose $3, 3\log_y x, 3\log_z y,$ and $7\log_x z$ are consecutive terms of an arithmetic sequence.
   a Prove that $x^{18} = y^{21} = z^{28}$.
   b Find $x$ in terms of: i $y$ ii $z$.

The following erratum was made on 19/Dec/2016

page 872  ANSWERS EXERCISE 2J 12 c, should have parts ii and iii correctly labelled and include domains on both sides:
12  b i is the only one
   c ii Domain = $\{x \mid x \leq 1\}$ or Domain = $\{x \mid x \geq 1\}$
   iii Domain = $\{x \mid x \geq 1\}$ or Domain = $\{x \mid x \leq -2\}$
The following erratum was made on 28/Feb/2018

Page 721 **SECTION 23C, VARIANCE AND STANDARD DEVIATION** should lose the second paragraph and read:

**VARIANCE AND STANDARD DEVIATION**

The problem with using the range and the IQR as measures of spread or dispersion of scores is that both of them only use two values in their calculation. Some data sets can therefore have their spread characteristics hidden when the range or IQR are quoted. We therefore turn to the **variance** and **standard deviation** of a data set.

Consider a data set of $n$ values: $x_1, x_2, x_3, ..., x_n$, with mean $\bar{x}$.

$x_i - \bar{x}$ measures how far $x_i$ deviates from the mean, so one might suspect that the mean of the deviations $\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})$ would give a good measure of spread. However, this value turns out to always be zero.

Instead, we define the **variance** of $n$ data values to be $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$.

Notice in this formula that:

- $(x_i - \bar{x})^2$ is also a measure of how far $x_i$ deviates from $\bar{x}$. However, the square ensures that each term in the sum is positive, which is why the sum turns out not to be zero.
- If $\sum_{i=1}^{n} (x_i - \bar{x})^2$ is small, it will indicate that most of the data values are close to $\bar{x}$.
- Dividing by $n$ gives an indication of how far, on average, the data is from the mean.

For a data set of $n$ values, $\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$ is called the **standard deviation**.

The square root in the standard deviation is used to correct the units. For example, if $x_i$ is the weight of a student in kg, $\sigma^2$ would be in kg$^2$. For this reason the standard deviation is more frequently quoted than the variance.

For the purpose of this course, we assume the formulae for variance and standard deviation of a whole population are the same as those for a sample.

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The following errata were made on 02/Nov/2016

Page 72 **EXAMPLE 15**, last line of solution should read:

$$1 \leq x < 2 \text{ or } x \geq 5.5$$

Page 660 **EXERCISE 21H**, advice about integration by parts should read:

When using ‘integration by parts’ the function $u$ should be easy to differentiate and the function $v'$ should be easy to integrate.
ANSWERS EXERCISE 2D 7 b, should read:

7 b \((g \circ f)(x) = 2x + 5, \ x \neq -2, \ \text{Domain} = \{x \mid x \neq -2\}\)

ANSWERS REVIEW SET 2B 13 c ii, should have y axis:

13 a \((g \circ f)(x) = \frac{2}{3x+1}\)

b \(x = -\frac{1}{3}\)

c i vertical asymptote \(x = -\frac{1}{3}\).

horizontal asymptote \(y = 0\)

ii

\[x = -\frac{1}{3}, \quad h(x) = \frac{2}{3x+1}\]

\((-3, -\frac{1}{3})\)

\((2, \frac{2}{5})\)

ANSWERS EXERCISE 14G 2 c and d, should state length of OP:

2 c

\[P(3,1,4)\]

\[P(-1,-2,3)\]

\[\text{OP} = \sqrt{26} \text{ units}\]

2 d

\[P(-1,-2,3)\]

\[\text{OP} = \sqrt{14} \text{ units}\]

ANSWERS EXERCISE 15H.1 6 b, should read:

6 b If \(k \neq 4\), the system is inconsistent and so has no solutions.

The lines are parallel.

If \(k = 4\), the system has infinitely many solutions of the form \(x = t, \ y = 3t - 2, \ t \in \mathbb{R}\). The lines are coincident.

ANSWERS EXERCISE 16F.2 4, Hint is for 4 b ii not 4 a ii :

4 b ii Hint: The LHS is a geometric series.

The following errata were made on 10/Jun/2016

EXERCISE 1H 11, should read:

11 \(b_1, c_1, b_2, \text{ and } c_2 \text{ are real, non-zero numbers such that } b_1 b_2 = 2(c_1 + c_2)\). Show that at least one of the equations \(x^2 + b_1 x + c_1 = 0, \ x^2 + b_2 x + c_2 = 0\) has two real roots.

SELF-INVERSE FUNCTIONS first dot point, should read:

For example:

- The function \(f(x) = x\) is the \textit{identity function}, and is also a self-inverse function.

EXERCISE 2J 14 b iv, should read:

14 a Explain why \(f(x) = x^2 - 4x + 3\) is a function but does not have an inverse function.

b i Explain why \(g(x) = x^2 - 4x + 3\) where \(x \geq 2\) has an inverse function.

ii Show that the inverse function is \(g^{-1}(x) = 2 + \sqrt{x+1}\).

iii State the domain and range of:

A \(g\)

B \(g^{-1}\).

iv Show that \((g \circ g^{-1})(x) = (g^{-1} \circ g)(x)\), the \textit{identity function}.

THE REMAINDER THEOREM first line, should clarify:

Consider the \textbf{real} cubic polynomial \(P(x) = x^3 + 5x^2 - 11x + 3\).
If \( z = \left( \frac{5}{2} - 3i \right)^3 \), express \( z \) in the form \( z = x + yi \), \( x, y \in \mathbb{Z} \).

We also observe that \( \text{cis} (-n\theta) = \text{cis} (0 - n\theta) = \frac{\text{cis } 0}{\text{cis } n\theta} \) \{as \( \frac{\text{cis } \theta}{\text{cis } \phi} = \text{cis} (\theta - \phi) \}\}

In Example 23 we showed that the cube roots of 1 are 1, \( w \), \( w^2 \) where \( w = \text{cis} \left( \frac{2\pi}{3} \right) \).
ERRATA
MATHEMATICS FOR THE INTERNATIONAL STUDENT
MATHEMATICS HL (Core) (3rd edition)

Third edition - 2013 first reprint

The following errata were made on or before 12/Jan/2015

page 43 Second dot point should read:

**PART G Problem solving with quadratics**

For two distinct real roots, \( \Delta > 0 \):
- \( k < -9 \) or \( k > -1, \ k \neq 0 \).

For two real roots, \( \Delta \geq 0 \):
- \( k \leq -9 \) or \( k \geq -1, \ k \neq 0 \).

page 26 Should read:

**EXAMPLE 10 Answers parts a and b**, should read:

**a** For two distinct real roots, \( \Delta > 0 \):
- \( k < -9 \) or \( k > -1, \ k \neq 0 \).

**b** For two real roots, \( \Delta \geq 0 \):
- \( k \leq -9 \) or \( k \geq -1, \ k \neq 0 \).

The following errata were made on 21/Oct/2015

page 952 Should read:

**ANSWERS REVIEW SET 25C 10 a ii**, should read:

\[
\begin{align*}
10 \ a \ & i \ \mu = 1.28, \ \sigma = 1.13 & \ b \ & \approx 0.366 \\
& ii \ E(Y) = 1.14, \ \sigma Y = 0.566
\end{align*}
\]

page 130 Proof at bottom of the page, should read:

- \( \log_c (AB) = \log_c (c^{\log_c A} \times c^{\log_c B}) = \log_c (c^{\log_c A + \log_c B}) = \log_c A + \log_c B \)
- \( \log_c \left( \frac{A}{B} \right) = \log_c \left( \frac{c^{\log_c A}}{c^{\log_c B}} \right) = \log_c \left( c^{\log_c A - \log_c B} \right) = \log_c A - \log_c B \)
- \( \log_c \left( A^n \right) = \log_c \left( (c^{\log_c A})^n \right) = n \log_c A \)

page 188 Division by quadratics, blue box at top of the page should read:

If \( P(x) = ax^2 + bx + c \) is divided by \( ax^2 + bx + c \) then

\[
\frac{P(x)}{ax^2 + bx + c} = Q(x) + \frac{ex + f}{ax^2 + bx + c}
\]

where \( ax^2 + bx + c \) is the divisor, \( Q(x) \) is the quotient, and \( ex + f \) is the remainder.

The remainder will be linear if \( e \neq 0 \), and constant if \( e = 0 \).
PERIODICITY OF TRIGONOMETRIC RATIOS

Since there are $2\pi$ radians in a full revolution, if we add any integer multiple of $2\pi$ to $\theta$ (in radians) then the position of $P$ on the unit circle is unchanged.

For $\theta$ in radians and $k \in \mathbb{Z}$,

$$\cos(\theta + 2k\pi) = \cos \theta \quad \text{and} \quad \sin(\theta + 2k\pi) = \sin \theta.$$  

We notice that for any point $(\cos \theta, \sin \theta)$ on the unit circle, the point directly opposite is $(-\cos \theta, -\sin \theta)$.

For $\theta$ in radians and $k \in \mathbb{Z}$, $\tan(\theta + k\pi) = \tan \theta$.

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RATES OF CHANGE

We often judge performances by rates. For example:

- Sir Donald Bradman’s average batting rate at Test cricket level was 99.94 runs per innings.
- Michael Jordan’s average basketball scoring rate was 30.1 points per game.
- Rangi’s average typing rate is 63 words per minute with an error rate of 2.3 errors per page.

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INVESTIGATION 3 4 b

Integrate $\frac{d\theta}{dx}$ with respect to $x$. Use $\theta(0) = 0$ to show that $\theta(x) = x$ for all $x$. 

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The following errata also apply to books printed in 2012

page 66  EXERCISE 2E 7 b, should read:

7 Prove that:

a. the sum of two even functions is an even function
b. the difference between two odd functions is an odd function
c. the product of two odd functions is an even function.

page 255  EXAMPLE 14 Answer part c, should read:

c. The total number of combinations
   = the number with 3 men and 1 woman + the number with 2 men and 2 women
   + the number with 1 man and 3 women
   = (3 choose 3) × (4 choose 1) + (3 choose 2) × (4 choose 2) + (3 choose 1) × (4 choose 3)
   = 665

page 267  THE PROCESS OF INDUCTION, at the bottom of the page should read:

The nth odd number is \((2n - 1)\), so we could also write the conjecture as:

\[1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1) = n^2 \quad \text{for all } n \in \mathbb{Z}^+\]

or \[\sum_{i=1}^{n} (2i - 1) = n^2 \quad \text{for all } n \in \mathbb{Z}^+.\]

page 275  EXERCISE 9B.3 2 a, should read:

2 Use the principle of mathematical induction to prove the following propositions:

a. \[3^n \geq 1 + 2n \quad \text{for all } n \in \mathbb{N}\]

b. \[n! \geq 2^n \quad \text{for all } n \in \mathbb{Z}, \quad n \geq 4\]

page 493  EXAMPLE 16, should say in the hint:

a. \[2z = 2\sqrt{2} \cos \theta\]

\[|2z| = 2\sqrt{2} \quad \text{and} \quad \text{arg}(2z) = \theta\]

The range of \text{arg} is \([-\pi, \pi\].

page 619  EXAMPLE 11, should say in the first line of the answer:

Let \(x\) cm be the lengths of the sides of the cube, so the surface area \(A = 6x^2\ \text{cm}^2\) and the volume \(V = x^3\ \text{cm}^3\).

page 809  REVIEW SET 25B 1, should read:

1 A discrete random variable \(X\) has probability distribution function \(P(x) = k \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{3-x}\)

where \(x = 0, 1, 2, 3\) and \(k\) is a constant. Find:

page 850  EXERCISE 27A 119 f, should read:

119 f The region bounded by \(y = f(x)\), the \(x\)-axis, and the line \(x = k\), \(k > 0\), has area \(\frac{1}{4}(e - 1)\) units\(^2\). Find the value of \(k\).
page 873 ANSWERS EXERCISE 2K 1 c v, should include all x-intercepts:

\[ y = f(x) \]

\[ x = -2.04 \text{ or } x = 1.71 \]

page 873 ANSWERS EXERCISE 2L 2, graph for \( y = f(x) \) should be:

\[ y = f(x) \]

\[ x = -2.04 \text{ or } x = 1.71 \]

page 874 ANSWERS EXERCISE 3F 6 d iii, should read:

\[ y \approx 2.62 \]

page 880 ANSWERS EXERCISE 4G 1 e, should read:

\[ y = 1 - 2 \log x \]

page 881 ANSWERS EXERCISE 4H 5, should read:

5 9 years

page 885 ANSWERS EXERCISE 5D 1 f, graphs should have x-intercept -1:

\[ y = -2(x+1) \]

\[ y = -2(x+1)^2 - 1 \]

page 885 ANSWERS EXERCISE 5E 1 d, should read:

1 d A reflection in the x-axis, then a vertical stretch with scale factor 2.

page 890 ANSWERS REVIEW SET 5B 11 c, should read:

11 b (1, 1) and (2, -1)  c VA is \( x = \frac{3}{2} \), y-int is \( \frac{1}{4} \)

page 894 ANSWERS REVIEW SET 6B 17 b, graph should have function:

\[ y = 2(x^2 - 2x + 5)(x^2 - 2x - 11) \]

\[ (1, -96) \]

\[ (-1, -126) \]

\[ (3, -128) \]
5. **Conjecture.** The number of regions for \( n \) points placed around a circle is given by \( C_n = 2^{n-1} - 1, \ n \in \mathbb{Z}^+ \).

Proof. The conjecture is illustrated by the figure. The first point makes a single region. Each additional point creates two new regions, one on each side of the circle. The number of regions formed by \( n \) points is then given by \( C_n = 2^{n-1} - 1 \), as the first point makes a single region and each subsequent point creates two new regions.

No. By the conjecture we expect \( 2^5 = 32 \) regions, but there are only 31.

2. **Hint:** \( \theta - \phi = - (\phi - \theta) \)

9. a maximum value \( 16 \) when \( x = 5 \)

b i \( y = 12 - x \)

ii \( y^2 = x^2 - (\cos 45^\circ) + 64 \)

2. a translation through \( \left( \frac{1}{2} \right) \)

b reflection in x-axis

c horizontal stretch, factor 2 and vertical stretch with factor 2

7. a \( y = \frac{3}{4} \sin (4x - 0.340) + \frac{1}{4} \)

b \( y = \tan (\pi - \frac{x}{2}) \)

5. a The plane's speed in still air would be \( 437 \) km/h.

b The wind slows the plane down to \( 400 \) km/h.

2. a 1.34 m/s in the direction \( 26.6^\circ \) to the left of her intended line.

b i \( 30^\circ \) to the right of Q

ii \( 1.04 \) m/s

3. a 24.6 km/h

b \( \approx 9.93^\circ \) east of south

4. a 82.5 m

b \( 23.3^\circ \) to the left of straight across

c 48.4 s

5. a The plane's speed in still air would be \( \approx 437 \) km/h.

b The wind slows the plane down to \( 400 \) km/h.

3. a 75.3°

b \( h_1 \cdot h_2 = 0 \)

3. \( 75.5^\circ \)
ANSWERS REVIEW SET 15B 6, should read:

- **a** \( x_1(t) = 2 + t, \ y_1(t) = 4 - 3t, \ t \geq 0 \)
- **b** \( x_2(t) = 13 - t, \ y_2(t) = [3 - 2a] + at, \ t \geq 2 \)
- **c** interception occurred at 2:22:30 pm
- **d** bearing \( \approx 12.7^\circ \) west of south, \( \approx 4.54 \) units per minute

ANSWERS EXERCISE 16B.1 4 **b**, should read:

- **a** \( |r| \)

ANSWERS EXERCISE 16C.1 3, should read:

- \( k\sqrt{7} \text{cis} \left( \frac{\pi}{4} \right) \) if \( k > 0 \), \( -k\sqrt{7} \text{cis} \left( -\frac{\pi}{4} \right) \) if \( k < 0 \),
- not possible if \( k = 0 \)

ANSWERS EXERCISE 19B 7, should read:

- **a** decreasing for \(-1 < x < 1\), never increasing
- **b** increasing for \(-1 < x < 1\), never decreasing

ANSWERS EXERCISE 19D.1 12 **b**, should read:

\[
f(t) = Ate^{-bt}
\]

ANSWERS REVIEW SET 19A 8 **c**, should read:

- **c** \( f'(x) \leq 0 \) for \( x < 1 \) and \( 1 < x \leq 2 \)
- and \( f'(x) \geq 0 \) for \( x \geq 2 \)

ANSWERS EXERCISE 20A.2 2 **a**, should read:

<table>
<thead>
<tr>
<th>( s(t) )</th>
<th>0</th>
<th>20</th>
<th>( t )</th>
<th>( v(t) )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>( t )</th>
<th>( a(t) )</th>
<th>0</th>
<th>20</th>
</tr>
</thead>
</table>

ANSWERS EXERCISE 20B 15, should read:

- \( \frac{\sqrt{L}}{4} \) cm² per radian

ANSWERS EXERCISE 20C 30, should read:

- **30** at grid reference \((3.54, 8)\), \( \approx 15.6 \) km

ANSWERS EXERCISE 22C.2 8, should read:

- **a** \( \nu(t) = \frac{1}{t + 1} - 1 \text{ m}\text{s}^{-1} \)
- **b** \( s(t) = \ln |t + 1| - t \) metres
- **c** \( s(2) = \ln 3 - 2 \approx -0.901 \) m, \( \nu(2) = -\frac{2}{3} \text{ m}\text{s}^{-1} \),
\( a(2) = -\frac{1}{2} \text{ m}\text{s}^{-2} \)
- The object is approximately 0.901 m to the left of the origin, travelling left at \( \frac{2}{3} \text{ m}\text{s}^{-1} \), with acceleration \(-\frac{1}{2} \text{ m}\text{s}^{-2} \).

ANSWERS REVIEW SET 24B 7 **a**, should read:

- \( 2 \text{ cm}^2 \)
3a \( X = 0, 1, 2, 3, \) or 4

3 a \( x \quad 0 \quad 1 \quad 2 \quad 3 \)

\( p_x \quad (1 - p)^x \quad 3px(1 - p)^2 \quad 3p^2(1 - p) \quad p^3 \)

1 b \( p_x = \frac{(1.5)^x e^{-1.5}}{x!} \) for \( x = 0, 1, 2, 3, 4, 5, 6, \ldots \)

The fit is excellent.

1 a \( k = \frac{2}{5} \) b \( 0.975 \) c \( 2.55 \) d \( \approx 0.740 \)